

# CIMPA School Final Report

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**Title of the School :** CIMPA Research School on Algebra and Algebraic Geometry for Applications

**Dates :** 26 February - 8 March 2024

**Location :** Lahore University of Management Sciences DHA Phase 5 Khayaban-e-Jinnah Road opposite Sector U Punjab Small Industries Housing Society, Lahore, Punjab 54792, Pakistan

## I. Summary

It was matter of pride for LUMS to host the CIMPA Research School on Algebra and Algebraic Geometry for Applications, scheduled to run from February 26 to March 8, 2024. The primary goal of this school is to provide students with a comprehensive understanding of modern algebraic and geometric methods, as well as their practical applications. The introductory courses will establish a solid foundation in recent developments such as combinatorial algebra, tropical combinatorics, and statistical models like graphical models. The more advanced lectures will focus on applied areas, such as dynamical systems, and explore their connections to algebra and geometry. The use of symmetries and other approaches to reduce complexity for high-dimensional phenomena will also be discussed. To further enhance their skills, participants will have access to training sessions and group discussions to delve deeper into the material and exchange ideas with peers. Renowned mathematicians from all over the World shared their expertise as speakers. The distinguished lineup includes.

- ) Prof. Thomas Kahle, Otto-von-Guericke Universitat Magdeburg, Germany
- ) Sara Saeedi Madani, Amir Kabir University of Technology and IPM, Iran
- ) Carlos Amendola, Technical University of Berlin, Germany
- ) Aida Maraj, from University of Michigan, USA
- ) Beatriz Pascual Escudero, Universidad Carlos III de Madrid, Spain
- ) Miruna-Stefana Sorea, "Lucian Blaga" University of Sibiu, Romania
- ) Cordian Riener, UiT - The Arctic University, Norway
- ) Imran Anwar and Shaheen Nazir, Lahore University of Management Sciences, Pakistan.

Participants from diverse countries, such as Iran, Vietnam, Canada, Germany, and Spain, convened at LUMS for this enriching academic experience. Additionally, a significant number of attendees joined the talks online. IBA Karachi co-hosted this CIMPA School with LUMS, attracting many participants from Karachi and Sindh to attend lectures at IBA. Throughout the School, speakers delivered lectures from both LUMS and IBA venues.

The planning of this CIMPA school was overseen by an esteemed scientific board comprising of:

- ) Prof. Tim Römer, Osnabrück University, Germany
- ) Prof. Alicia Dickenstein, the University of Buenos Aires, Argentina
- ) Prof. Le Tuan Hoa, the Vietnam Institute for Advanced Study in Mathematics, Vietnam
- ) Prof. Sara Saeedi Madani , Amir Kabir University of Technology and IPM, Iran;
- ) Prof. Shaheen Nazir and Prof. Imran Anwar from LUMS, Lahore.

This CIMPA School presented a great opportunity for Pakistani students and researchers to engage with international peers, discussing the role of algebra and geometry in data analysis and graphical models.

LUMS was thrilled to host the CIMPA Research School on Algebra and Algebraic Geometry for Applications during February 26 – March 08, 2024 fostering collaboration and knowledge exchange among mathematicians worldwide. We extended our heartfelt gratitude to the esteemed international sponsors for their generous financial support, including CIMPA, ICTP (Italy), the International Mathematical Union (IMU), and the European Mathematical Society (EMS). Furthermore, our gratitude extends to the Higher Education Commission (HEC) of Pakistan, Syed Babar Ali School of Science and Engineering (SBASSE), Suleman Dawood School of Business (SDSB), and LUMS Syed Ahsan Ali and Syed Maratib Ali School of Education (SOE) for their generous support in making this event possible.

This CIMPA School not only solidifies LUMS's reputation as a stronghold for fostering a research culture in mathematics but also served as a catalyst for regional and international research networking and collaborations. By bringing together experts and enthusiasts from diverse backgrounds and regions, this event paved the way for fruitful exchanges of ideas, collaborative projects, and lasting partnerships. It marked a significant stride towards the advancement of mathematical research and education, resonating with the spirit of a famous quote by one of the greatest mathematicians, David Hilbert: *'Mathematics knows no races or geographic boundaries; for mathematics, the cultural world is one country.'* This event embodied the ethos of collaboration and inclusivity, fostering a unified global community in pursuit of mathematical excellence.

## **II. Scientific content**

The Department of Mathematics at the School of Science and Engineering hosted one of the most prestigious research schools on mathematics CIMPA Research School Algebra and Algebraic Geometry for Applications

It was an intensive research school on the research topics that evolved during the recent years. Here is a short description of each course delivered during the CIMPA School.

### **Shaheen Nazir (week 0, LUMS Lahore) Feb 19 – 22 2024**

It was an intensive immersion into the realm of Algebraic Geometry, delving deep into both affine and projective geometries through an algebraic lens. Commencing with fundamental principles such as the Hilbert Nullstellensatz theorem, the course traversed through intricate concepts and techniques, culminating in an exploration of toric varieties. From unraveling the foundational structures to navigating the complexities of geometric spaces, participants were equipped with a comprehensive understanding of Algebraic Geometry's rich tapestry, offering insights into both theoretical underpinnings and practical applications.

### **Thomas Kahle (Otto-von-Guericke Universitat Magdeburg, Germany)**

This course provided a deep dive into the cutting-edge realm of algebraic statistics, a field that has recently witnessed significant evolution. Through a novel approach, Markov chains were scrutinized with a focus on their connection to lattice points. Led by Thomas, participants were introduced to the innovative framework of toric algebra, where computational algebraic and combinatorial tools were skillfully wielded to uncover insights. Encompassing a wide array of topics, the course extensively explored the application of algebraic geometry in the realm of data science, showcasing its relevance and efficacy in solving real-world problems. From theoretical foundations to practical methodologies, attendees gained a comprehensive understanding of how algebraic techniques can be leveraged to extract meaningful patterns and information from data.

### **Beatriz Pascual Escuardo (Universidad Politécnica de Madrid, Spain)**

This course delved into the fascinating intersection of algebra and chemistry through the lens of Chemical Reaction Networks (CRNs). The instructor provided an illuminating exploration of the algebraic underpinnings inherent in the dynamics of CRNs, offering valuable insights into the interconnectedness of these two seemingly disparate fields. Participants were guided through a comprehensive analysis of the dynamical systems associated with CRNs, uncovering the underlying algebraic structures that govern their behavior.

Throughout the course and accompanying tutorials, the instructor adeptly elucidated the intricacies of toric algebra, equipping attendees with the computational and theoretical tools necessary to navigate this complex terrain. Through a series of engaging examples, participants gained a deep understanding of how toric algebra can be applied to elucidate the dynamics of chemical reactions, paving the way for innovative approaches in chemical kinetics and reaction network analysis. Whether exploring theoretical concepts or tackling practical challenges, the course provided a robust foundation for further exploration and application in the realm of chemical sciences.

#### **Miruna-Stefana Sorea ("Lucian Blaga" University of Sibiu, Romania)**

This course delved into the captivating realm of Disguised Dynamical Systems, characterized by their intriguing and often elusive properties. Through a meticulous examination, the instructor highlighted the intricate interplay between nonlinear dynamical systems and nonlinear algebra, showcasing the profound impact of their integration on various fields, particularly in biochemical dynamics and systems biology.

Participants were immersed in an exploration of the multifaceted nature of disguised dynamical systems, unraveling their complex behaviors and underlying mathematical structures. By delving into the fundamental principles that govern these systems, attendees gained a deeper appreciation for their significance in modeling real-world phenomena, such as biochemical processes and biological systems.

The course not only provided a theoretical foundation but also offered practical insights into the application of disguised dynamical systems in addressing challenges in system biology. Through a combination of theoretical discussions, hands-on exercises, and real-world examples, participants were equipped with the tools and knowledge to navigate the complexities of these systems, paving the way for further advancements and innovations in the field.

#### **Carlos Amendola (Technical University of Berlin, Germany)**

The course offered a captivating exploration into the realms of likelihood geometry and Bayesian network analysis, showcasing their diverse applications in probabilistic data studies. Carlos adeptly navigated through the intricate landscapes of algebraic and combinatorial tools, unveiling their profound relevance in the realm of multivariate statistical models.

Through a series of engaging presentations, participants were introduced to a plethora of captivating results and astonishing applications stemming from the fusion of algebraic geometry with statistical studies. Carlos masterfully elucidated the underlying principles and methodologies, providing attendees with a comprehensive understanding of how these tools can be leveraged to unravel complex patterns and relationships within datasets.

From uncovering hidden structures to extracting meaningful insights, the course illuminated the transformative potential of algebraic geometry in statistical analysis. By showcasing real-world applications and demonstrating practical techniques, participants gained invaluable insights into harnessing these powerful tools to address contemporary challenges in data analysis and interpretation. Whether exploring theoretical frameworks or delving into practical applications, Carlos' course provided a rich and insightful journey into the intersection of algebraic geometry and statistical studies.

#### **Sara Madani (Amirkabir University of Technology and IPM, Iran)**

Sara's presentation delved into the captivating topic of the Matching power of monomial ideals, unveiling its rich tapestry of combinatorial and algebraic properties. Throughout the session, Sara expertly navigated through a myriad of intriguing results and properties, shedding light on the fascinating interplay between combinatorial structures and algebraic ideals.

Of particular note were the discussions surrounding normalized depth, a concept central to Sara's recent collaborative work with Erey, Herzog, and Hibi. Sara provided valuable insights into the theoretical underpinnings of normalized depth, offering a deep understanding of its significance within the context of monomial ideals. Moreover, she skillfully presented open problems stemming from this line of research, highlighting avenues for future exploration and discovery in the field.

By intertwining theoretical rigor with practical relevance, Sara's presentation not only enriched participants' understanding of matching power ideals but also inspired further inquiry into related areas of study. Her adept exposition of complex concepts and cutting-edge research served to foster a vibrant intellectual discourse, fueling innovation and advancement within the realm of combinatorial and algebraic theory.

### **Imran Anwar (LUMS, Pakistan)**

The course provided a comprehensive exploration of algebra retraction and its burgeoning connections with the Zariski Cancellation Problem, a central enigma in algebraic geometry first conjectured by O. Zariski. Delving into the intricacies of this fundamental problem, participants were immersed in a deep dive into the study of algebraic retracts from a combinatorial perspective.

Central to the discourse were the innovative concepts surrounding cut algebras, a groundbreaking development introduced by Strumfels and Sullivant in recent years. Through a meticulous examination of these cutting-edge constructs, attendees gained valuable insights into the underlying combinatorial structures and algebraic properties that underpin algebraic retraction.

The course not only elucidated the theoretical foundations but also showcased practical applications and implications of these concepts in tackling longstanding conjectures and open problems in algebraic geometry. By fostering a dynamic exchange of ideas and insights, participants were empowered to engage with advanced topics at the intersection of algebra and combinatorics, paving the way for further advancements in both fields.

### **Aida Maraj (University of Michigan, USA)**

The course delved into the intricate realm of symmetric Noetherian structures, shedding light on their fascinating properties and connections to various algebraic phenomena. Through an insightful presentation, attendees were introduced to a range of intriguing results centered on the Equivariant Hilbert series of these structures, unraveling their underlying symmetries and algebraic intricacies.

Of particular interest were the conjectures surrounding the asymptotic behavior of algebraic properties, which served as focal points for further exploration and inquiry. By probing into these conjectures, participants gained valuable insights into the long-term trends and patterns exhibited by symmetric Noetherian structures, offering a deeper understanding of their algebraic nature.

The course not only provided a rigorous theoretical framework but also encouraged active engagement with open problems and conjectures, fostering a collaborative environment for intellectual exchange and discovery. Through a combination of theoretical discussions and practical examples, attendees were equipped

with the tools and knowledge to tackle complex algebraic problems related to symmetric Noetherian structures, paving the way for further advancements in the field.

### **Cordian Reiner (UiT - The Arctic University, Norway)**

Cordian's course delved deeply into the heart of the invariant theory of commutative algebra, drawing inspiration from the groundbreaking work of David Hilbert and his seminal Hilbert bases theory. Throughout the sessions, Cordian skillfully navigated through a myriad of central problems in this rich field, offering participants a comprehensive exploration of both classical and modern perspectives.

By expanding the spectrum of inquiry, Cordian unveiled a treasure trove of interesting and important results, each revealing the deep intricacies inherent in the study of invariant theory. From classical foundational concepts to cutting-edge developments, attendees were exposed to a diverse array of topics, providing a holistic understanding of the field's evolution and significance.

Through a combination of theoretical exposition, problem-solving sessions, and interactive discussions, Cordian fostered a dynamic learning environment that encouraged participants to engage deeply with the material. By showcasing the breadth and depth of invariant theory, Cordian's course not only honored the legacy of David Hilbert but also inspired new avenues of research and exploration in commutative algebra and beyond.

One may fetch more information and details about these courses and projects by visiting the website <https://sites.google.com/view/cimpa-lums2024/home> which has video recordings of lectures, lecture notes, and important references.

### III Participant List

List here all participants, including local participants, foreign participants without CIMPA support and foreign participants supported by CIMPA. Please mention possible strong participants (with their email) that you think CIMPA should follow (CIMPA wishes to serve as recommendation center for students applying to doctoral or postdoctoral grants. Also these persons may be natural contacts for CIMPA in the future).

Participant List		
(CIMPA Research Event February 26- March 08, 2024)		
Sr No:	Full Name:	Affiliation/Institute
1	Abbas Nasrollah Nejad*	Institute for Advanced Studies in Basic Sciences (IASBS) 444 Prof. Yousef Sobouti Blvd., Zanjan 45137-66731, <a href="mailto:abbasnn@iasbs.ac.ir">abbasnn@iasbs.ac.ir</a>
2	Abderrahmane Labane	Lahore University of Management Sciences (LUMS)
3	Abdul Basit	Lahore University of Management Sciences (LUMS)
4	Abdullah Ahmed*	Lahore University of Management Sciences (LUMS), <a href="mailto:24100035@lums.edu.pk">24100035@lums.edu.pk</a>
5	Abdur Rauf	Air University, Multan
6	Aleena	Lahore University of Management Sciences (LUMS)
7	Ali Haidar	International Mathematics Master
8	Ambreen Ahmed	University of Narowal
9	Ameer Hamza	Lahore University of Management Sciences (LUMS)
10	Ammara Rashid*	Lahore University of Management Sciences (LUMS), <a href="mailto:amara2799@gmail.com">amara2799@gmail.com</a>
11	Amna Farooq	Lahore University of Management Sciences (LUMS)
12	Anam Rani	CASPAM (Centre for Advanced Studies in Pure and Applied Mathematics)
13	Aniqa Ahmad	University of Central Punjab, Lahore.
14	Aqib Javed	COMSATS University Islamabad Park Road, Tarlai Kalan, Islamabad, Pakistan
15	Aqsa Noreen*	Lahore University of Management and Sciences LUMS, <a href="mailto:22070005@lums.edu.pk">22070005@lums.edu.pk</a>
16	Arhum Naseem Khawaja	Lahore University of Management Sciences (LUMS)
17	Asia Rauf	Government College Women University Faisalabad
18	Awais Shaukat*	Namal University, Mianwali, <a href="mailto:awais.shaukat@namal.edu.pk">awais.shaukat@namal.edu.pk</a>
19	Ayesha Andalib Kiran	COMSATS university of Islamabad, Lahore campus
20	Ayman Alyas	Abdus Salam School of Mathematical Sciences, GC University, Lahore
21	Azhar Javed	Lahore University of Management Sciences (LUMS)
22	Bac Trong Nguyen	Duy Tan University, Da nang city, Vietnam Department of Physical Sciences and Mathematics,
23	Faira Kanwal Janjua	Forman Christian College, University, Lahore
24	Faryal Chaudhry	The University of Lahore
25	Farzan William	COMSATS University Islamabad, Lahore Campus
26	Hafiz Muhammad Bilal	COMSATS University Islamabad, Lahore Campus Pakistan
27	Hafsa Bibi	Government College for Women University Faisalabad.
28	Hafsa Hafeez*	Lahore University of Management Sciences (LUMS), <a href="mailto:hafsahafeez2018@gmail.com">hafsahafeez2018@gmail.com</a>
29	Humaira Arshad Butt	International Mathematics Masters by ICTP.
30	Ijaz Jamil	Abdus Salam School of Mathematical Sciences, GC University, Lahore
31	Kamran Shakoor*	Govt. Islamia Graduate College, Kasur, <a href="mailto:kamranshakoor@sms.edu.pk">kamranshakoor@sms.edu.pk</a>
32	Khurram Shabbir	Government College University Lahore
33	M Awais Umar	Govt. Associate College Sharaqpur Sharif
34	Maria Naseem	University of Central Punjab

35	Masood Ahmad	Lahore University of Management Sciences (LUMS)
36	Mehtab Ketabi	Institute for Advanced studies in Basic Sciences (IASBS), P. O. Box 45195–1159, Gava Zang Street, Zanjan, Iran
37	Misbah Farheen	Namal University Mianwali
38	Mughees Ghayas*	Lahore University of Management Sciences (LUMS), 21070004@lums.edu.pk
30	Muhammad Bin Nasir	Abdus Salam School of Mathematical Sciences, GC University, Lahore
40	Muhammad Din	Abdus Salam School of Mathematical Sciences, GC University, Lahore
41	Muhammad Fazeel Anwar	Sukkur IBA University
42	Muhammad Izhar*	Government Post Graduate College Mardan Khyber Pakhtunkhwa Pakistan, mizharmath@gmail.com
43	Muhammad Javaid Qayoom*	Lahore University of Management Sciences (LUMS), 23070014@lums.edu.pk
44	Muhammad Kamran Siddiqui	COMSATS University Islamabad, Lahore Campus, Pakistan.
45	Muhammad Naeem	National University of Sciences and Technology (NUST), Islamabad
46	Muhammad Naeem Aslam	Pakistan Institute of Engineering & Applied Sciences (PIEAS)
47	Muhammad Saqib	Lahore University of Management Sciences (LUMS)
48	Muhammad Shoaib Khan*	Lahore University of Management Sciences (LUMS), 22070004@lums.edu.pk
49	Muhammad Toheed Jillani*	Quaid-i-Azam University Islamabad, muhammadtoheedjillani@gmail.com
50	Muhammad Waqar Bashir	COMSATS University Islamabad
51	Munawar Khan	Lahore University of Management Sciences (LUMS)
52	Muneeba Mansha	Comsats Institute of information technology Islamabad, Lahore Campus
53	Musab Umair Tahiri	Abdus Salam School of Mathematical Sciences, GC University, Lahore
54	Nam Nguyen Thi	University of the Philippines Manila, Department of Mathematics,
55	Nimra Javed*	Lahore Leads University Lahore, nimrajaved16@gmail.com
56	Nimraa Shoket	Abdus Salam School of Mathematical Sciences, GC University, Lahore
57	Rao Muhammad Touqeer	Quaid-i-Azam University Islamabad
58	Razzaq Junaid	University of Bologna, Italy
59	Rida Irfan	COMSATS University Islamabad, Sahiwal Campus
60	Rizwan Jahangir*	Sabancı University Orta Mah. Tuzla 34956, Istanbul, rizwan@sabancı-univ.edu
61	Sabeel Shamin	Government College University Lahore (GCUL)
62	Saeed Ahmed*	Govt Graduate College Model Town Lahore, sa_658@yahoo.com
63	Saliha Fatima	Namal university Mianwali
64	Sami Halaseh*	Free University Berlin, Germany, samihalaseh14@pm.me
65	Shafiq ur Rehman	COMSATS University Islamabad (Attock Campus)
66	Siraj Ul Haq	Lahore University of Management Sciences (LUMS)
67	Sophia Bugarija*	Berlin Mathematical School & Humbolt University, Germany, sophiabugarija@pm.me
68	Tehmina Qayyoom	Abdus Salam School of Mathematical Sciences, GC University, Lahore
69	Um e Aimen	GC Women University Faisalabad
70	Waqar Afzal	Government College University Lahore
71	Yazdan Pour Ali Akbar *	Institute for Advanced Studies in Basic Sciences (IASBS) No. 444, Prof. Yousef Sobouti Blvd. P.O. Box: 45195-1159, Zanjan 45137-66731, Iran, ali-yazdanpoor@gmail.com
72	Zill e Shams*	Department of Mathematics, The Women University, Multan, zilleshams.6376@wum.edu.pk
73	Zoha Saleem*	Lahore University of Management Sciences (LUMS), zo-hajutt1902@gmail.com
74	Zunaira Kosar	University of Sialkot

The highlighted one's are international participants, who got the CIMPA support.

\*The good active participant with their emails for future recommendations. These participants even continued with the questioning answering after the conclusion of the school.

#### IV. Financial Report

Provide here the list of all grants received for the organisation of your school.

CIMPA	€ 15,000	~ PKR 4,522,680
ICTP	€ 2,500	~ PKR 750,000
IMU-CDC	€ 1,500	PKR 442,784
EMS-CDC	€ 1,000	PKR297,231

Budget Head	Actual Expense	CIMPA	ICTP	IMU-CDC	EMS-CDC
Accommodation	1,912,188	422,173	750,000	442,784	297,231
Meal Expense	842,930	842,930	-	-	-
Printing & Stationery	808,350	808,350	-	-	-
IT Support/Logistics	350,000	350,000			
Air ticket (Lahore to Karachi for four in <b>three international Speakers</b> )	75,699	75,699			
Air Ticket (Dr. Sara Saeedi Madani, Iran - International Speaker)	352,162	352,162	-	-	-
<b>Total in PKR</b>	<b>4,341,329</b>	<b>2,851,314</b>	<b>750,000</b>	<b>442,784</b>	<b>297,231</b>
<b>Total in Euros</b>	<b>14,361.12</b>	<b>9,432.15</b>	<b>2500</b>	<b>1500</b>	<b>1000</b>

Then give details about expenses made with CIMPA funding.

Budget Head	Requested Funds from CIMPA	CIMPA Utilization
Accommodation	422,173	422,173
Meal Expense	842,930	842,930
Printing & Stationery	808,350	808,350
IT Support/Logistics	350,000	350,000
Air ticket (Lahore to Karachi for <b>Three international Speakers</b> )	75,699	75,699
Air Ticket (Dr. Sara Saeedi Madani, International Speaker from Iran)	352,162	352,162
Entry Tickets City Tour @1800 each 60 persons		108,000
Dinner with International speakers/participants		185,488
Transport charges		71,337
	-	-
<b>Total in PKR</b>	<b>2,851,314</b>	<b>3,216,139</b>
<b>Total in Euros:</b>	<b>9,432.15</b>	<b>10,639</b>

Additional Expense requested from CIMPA	
Expense Description	Amount in Euros
Air Ticket of Ms. Nam (PKR 486,321)	1,606.60
E-VISA fee for Iranian participants (due to issue of online payment) (PKR31,212)	103.11
<b>Total:</b>	<b>1,709.71</b>

Expenses without invoices	
Expense Description	Amount in Euros
Tour Guide (PKR 5,000)	17
Hotel Tip (PKR 2,000)	7
<b>Total:</b>	<b>24</b>

<b>Total Funds requested to CIMPA in Euros</b>	<b>12,373</b>
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## V. Pictures

*Please include pictures of school and social activities.*

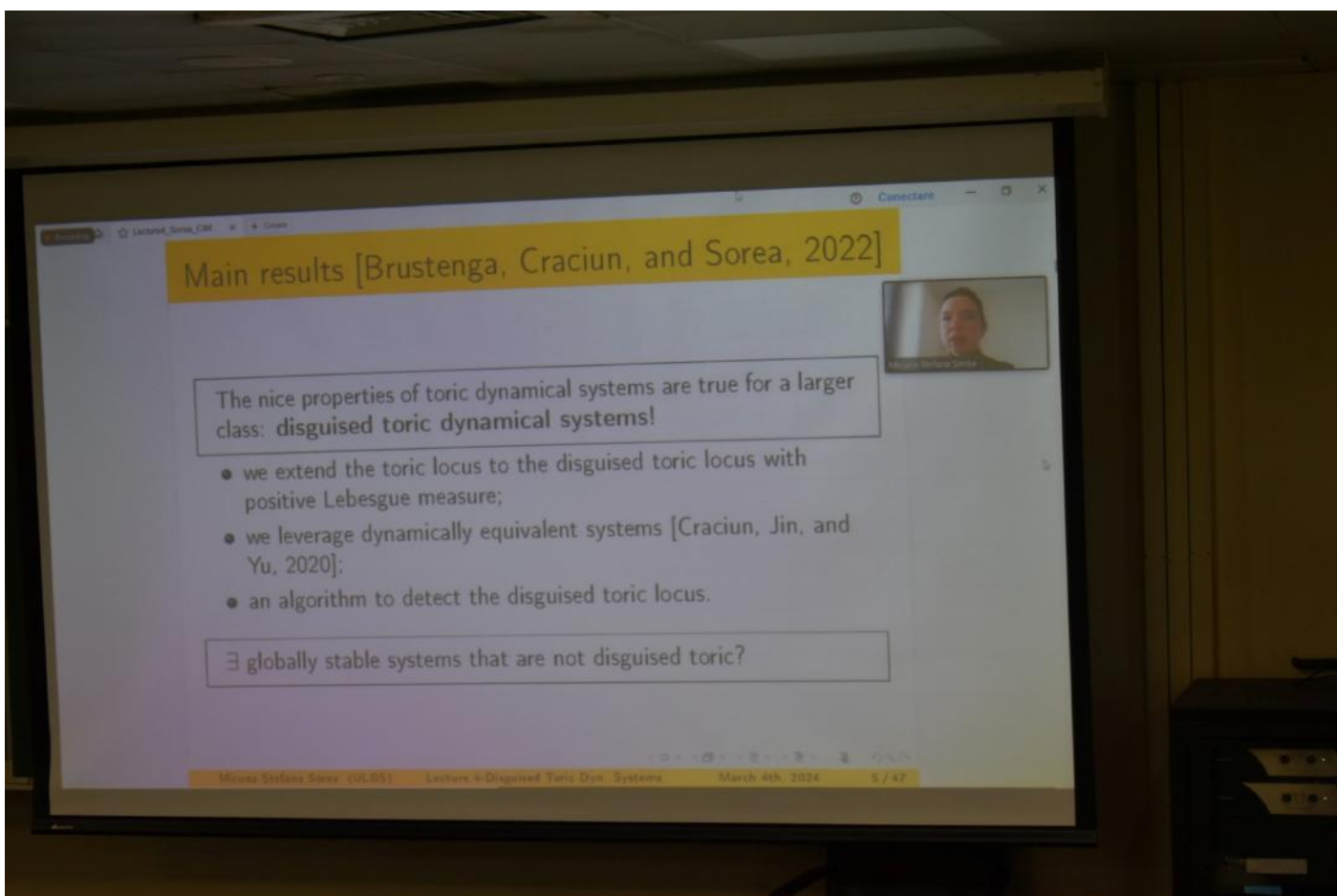
**From intense study sessions to engaging discussions, esteemed instructors had been guiding participants through a diverse array of mathematical topics, fostering collaboration and innovation every step of the way.**

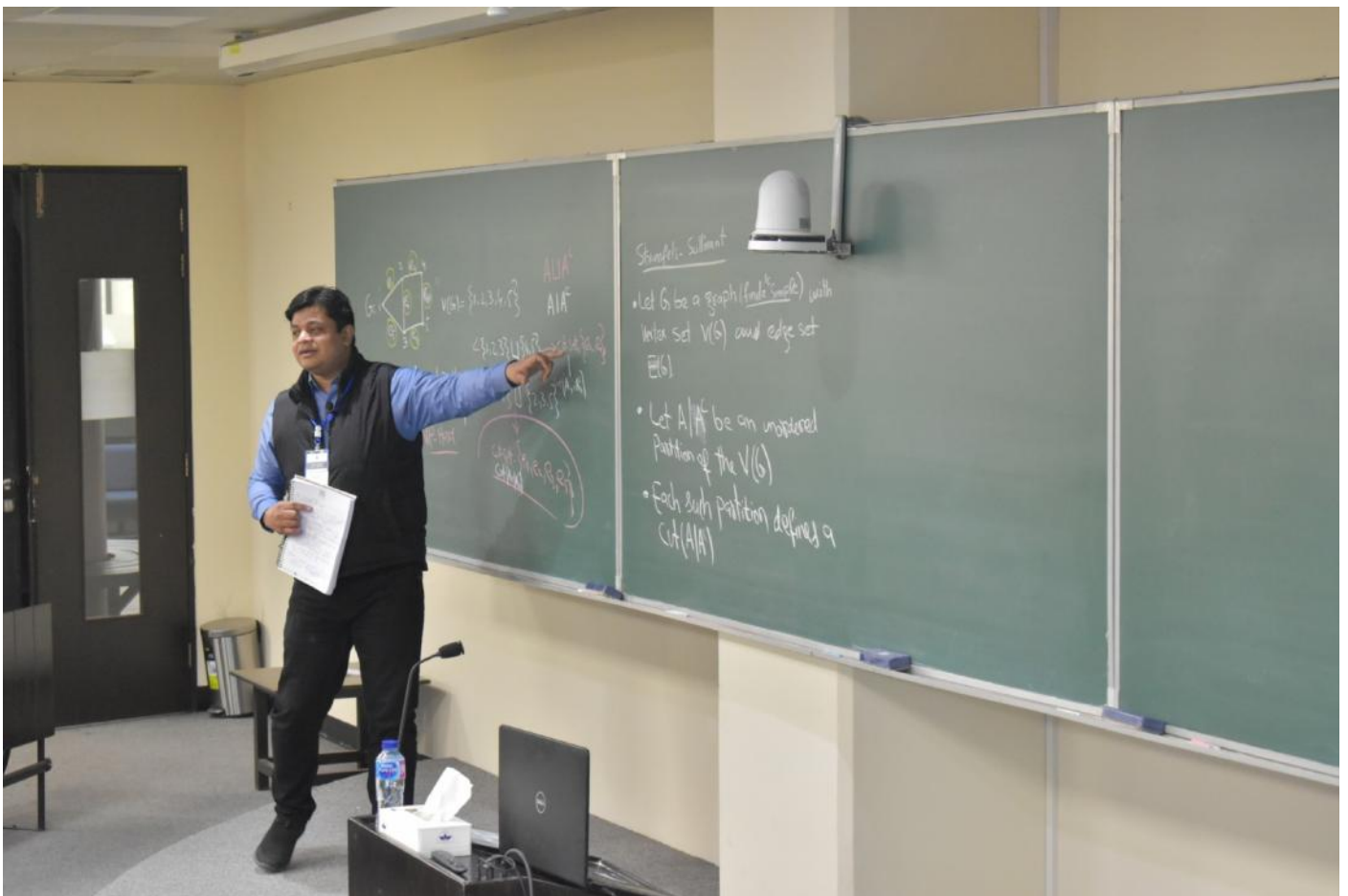
















**Tea Time Delights:** Sipping on steaming cups of tea and indulging in delectable treats, participants gathered around to unwind, share stories, and forge new friendships amidst the cozy ambiance of tea breaks.

**Laughter and Conversation:** Amidst the hustle and bustle of busy schedules, the participants took a moment to pause, connect, and enjoy each other's company, sharing laughter, engaging in meaningful conversations, and creating memories that would last a lifetime.







**Birthday Bash:** Surrounded by laughter and good cheer, the participant came together to celebrate the birthdays of their fellow, showering them with warm wishes, thoughtful gifts, and, of course, a delicious cake to mark the occasion!



**Celebrating International Women's Day at CIMPA School!**





As a token of gratitude and souvinere, TUGHRAS were presented to in-person speakers and CIMPA - Centre International de Mathématiques Pures et Appliquées scientific officer.









**Explore, Learn, and Connect: The CIMPA School City Tour Experience!**





