# TOPICS ON MONOMIALS AND POLYMATROIDAL IDEALS

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ABSTRACT. Let $R = K[x_1, \ldots, x_n]$ be a polynomial ring in $n$ variables over a field $K$ and $I$ be a monomial ideal of $R$. Let $\operatorname{astab}(I)$ and $\operatorname{dstab}(I)$ be the smallest integers $l$ and $k$, for which $\operatorname{Ass}(I^l)$ and $\operatorname{depth}(R/I^k)$ stabilize, respectively. In this presentation, the goal is to introduce and study some basic concepts from combinatorial commutative algebra. In particular, we concentrate on property of polymatroidal ideals and $\operatorname{astab}(I)$ and $\operatorname{dstab}(I)$. For each unexplained notion or terminology, we refer the reader to [8] and [20].

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