FREE BOUNDARY CONSTANT MEAN CURVATURE HYPERSURFACES

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Abstract: Differential Geometry is an area of pure mathematics that studies geometric properties of curves, surfaces, or in a more general context, manifolds. Such study involves the application of techniques and concepts from different branches of mathematics such as Analysis, Topology, Algebra, Partial Differential Equations, etc. In addition, Differential Geometry has a strong relationship with Physics (e.g. theory of relativity), Computer Graphics (e.g. animation), Industry (e.g. construction of helical parts) among others. In this mini-course, we are going to present a brief introduction to the study of constant mean curvature (CMC) free boundary hypersurfaces. Our goal is to provide the student with the necessary tools for understanding some classic results on CMC surface theory and to present an overview of the latest advances in the free boundary CMC hypersurfaces subject. To do this, first we present the basic concepts of Differential Geometry on hypersurfaces: shape operator, mean curvature, geodesic curvature, convexity, first and second variation of area and stability. Then, we are going to present classic results on CMC surfaces: Hopf's theorem, Alexandrov's theorem and Barbosa-do Carmo's theorem. After, we are going to explain some results involving stability or gaps conditions for problems of free boundary hypersurfaces in the ball. Finally, we comment on some open problems in this context.

Prerequisites: Background in Differential Geometry.

MSC: 53C20, 53A10, 49Q10

Keywords: mean curvature, free boundary, stability.

Duration: 8 hours with 2 hours per day.

Idiom: English.

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Schedule

- Day 1 We present some basic concepts of Differential Geometry: shape operator, mean curvature, geodesic curvature, convexity, first and second variations of the area.
- Day 2 We comment the classical Hopf's theorem [8] and Alexandrov's theorem [1] on CMC surfaces. Then we present the definition of stability and Barbosa-do Carmo's theorem [7].
- Day 3 We present some results on free boundary CMC hypersurfaces: Ros and Vergasta [10]; Ambrozio and Nunes [2]; Barbosa, Cavalcante and Pereira [5].
- Day 4 Finally, we discuss some latest advances on free boundary CMC hypersurfaces theory, for example: Andrade, Barbosa and Pereira [3]; Nunes [9]; Barbosa [4] and Barbosa and Espinar [6]. We finish presenting some open problems on this topic.

References

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