

"A (modest) Introduction to Fukaya Categories"
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Maxim Kontsevich in 1994, proposed the Homological Mirror Symmetry (HMS) conjecture at the ICM in Zurich, which is a homological version (meaning equivalence at the level of categories) of the phenomenon of Mirror Symmetry. Mirror Symmetry first observed by Physicists studying String Theory, serves as an unexpected bridge between the complex geometry of one manifold X and the symplectic geometry of its mirror manifold \check{X} . It has since inspired and driven important research in both Symplectic and Algebraic Geometry alike. The Fukaya categories constructed in the works of Fukaya and others are an essential piece of the HMS puzzle. These are fairly complicated entities which constitute the A-side (or the symplectic side) of the HMS conjecture of Kontsevich.

This course will be a gentle (and somewhat biased) introduction to Fukaya categories. Roughly speaking, Fukaya category associated to a symplectic manifold, is a category whose objects are Lagrangian submanifolds and whose morphisms are given by Floer chain complexes generated by intersections between these Lagrangian objects. These can be defined in various flavours depending on the geometric context at hand. The aim is to present as an example the topological Fukaya category of a closed surface of genus higher than one, using elementary geometry of curves on a surface, while suppressing the area form. Some familiarity with the language of categories may be helpful.

In this course we will include an overview of categories, their equivalence, A_∞ -categories and Floer theory. To make this beautiful yet intimidating area of mathematics interesting for a wider audience, and if time permits, we will present a categorification of the Burau representation of the Braid group by Bouchair, based on ideas from Khovanov-Seidel's categorification of the Braid group action on some derived category of modules.

Domain: Geometric Topology # Symplectic Geometry # Homological Algebra

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