Topological semantics for modal logic

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Topological semantics for modal logic was introduced by McKinsey and Tarski in the 1940s. There are two kinds of topological interpretations for modal operators proposing two different semantics: c-semantics and d-semantics. In this course, we first consider the c-semantics, in which the modal operator $\Diamond$ is interpreted as the closure operator. We provide examples showing that some topological properties are definable in modal logic. Then we discuss the idea of proving some significant completeness results. For example, we show that the logic of all topological spaces, as well as the real line, is $\mathsf{S4}$ with respect to the c-semantics. Then we investigate the d-semantics, in which $\Diamond$ is interpreted as the derived-set operator. With some examples, we see that the d-semantics is more expressive than the c-semantics. We review a proof showing that the d-logic of all topological spaces is $\mathsf{wK4}$. We also discuss the landscape of the d-logics over some interesting classes of topological spaces, such as Stone spaces. Finally, we speak about the Gödel-Löb provability logic, which is complete with respect to scattered spaces and any ordinal $\alpha \geq \omega^\omega$ equipped with the interval topology.