

CIMPA-ICTP Fellowships 2022 "Research in Pairs"

COURSE PROJECT:

Representation theory of generalized small quantum groups

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Research area: # Quantum Algebra # Representation Theory # Rings and Algebras

2010 Mathematics Subject Classification: # 16T05 # 16T20 # 16G99 # 20G42 # 16P10 # 20G05 # 17B10 # 22E47

Keywords: # Hopf algebras # Nichols algebras # Drinfeld doubles # Quantum groups # Representation theory # Triangular decomposition

Number of hours: 8.

Description: We will begin the course by exploring the very rich Representation Theory of the graded finite-dimensional algebras with triangular decomposition. We will follow mainly [1]. These algebras have plenty of powerful and beautiful methods and results inspired in Lie Theory (characters, Verma module, BGG Reciprocity, etc).

Then we will introduce some examples of Hopf algebras with triangular decomposition based on [2]. Among these, we will focus on the Drinfeld doubles of the bosonization of a finite-dimensional Nichols algebra over a semisimple Hopf algebra. We will investigate peculiar features which appear in this setting (duality and tensor product).

The structure of the course will be as follows.

(1) Part I:

- (a) Definition of algebras with triangular decomposition.
- (b) Standard and costandard modules. Definition and properties.
- (c) The classification of the simple modules.
- (d) On the characters of simples modules.
- (e) Brauer and BGG Reciprocities.

(2) Part II:

- (a) Definition of Nichols algebras.
- (b) A method to construct Hopf algebras with triangular decomposition
- (c) Example 1: Drinfeld doubles.
- (d) Example 2: Small quantum groups.
- (e) Duality of standard modules.
- (f) The tensor product between standard and costandard modules.

REFERENCES

- [1] G. Bellamy and U. Thiel. *Highest weight theory for finite-dimensional graded algebras with triangular decomposition*. Adv. Math. 330 (2018), 361 – 419.
- [2] C. Vay, *On Hopf algebras with triangular decomposition*, Contemp. Math. **728** (2019), 181-199.