Introduction to Optimization and Operator Theory

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December 5-8, 2022

Keywords: Differentiability, Gradient Operators, Duality, Duality and Optimization, Proximal Operator, Numerical Optimization.

Abstract: An introductory lecture on operator and optimization theory will be given. To be specific, various notions of derivatives for mappings between Normed spaces will be discussed, with a special emphasis on mappings taking values in $\mathbb{R}$ connecting derivatives with optimization problems. We discuss the continuity, lower semicontinuity, differentiability, and, most importantly, optimization properties of convex functions in normed spaces. We will also go over the Fenchel conjugate and biconjugate for convex functions, as well as their properties. With these tools at our disposal, we move on to the duality methods in optimization, which allows us to solve unconstrained and constrained minimization problems (called the primal) by relating them to their dual problems obtained via the Fenchel conjugate. Finally, after discussing the Moreau proximity operator in Hilbert spaces, we will look at a popular class of optimization algorithms.

References:

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