2020 CIMPA Research School
Project Proposal

I. General Information

Research School Title: Dynamical Systems and Applications
Official language of the school: English
Country: Benin
Name and address of the host institution (university or equivalent):
Institut de Mathématiques et de Sciences Physiques (Dangbo), Université d'Abomey-Calavi

Dates (option A): 29/06/2020-10/07/2020
Dates (option B): 16/11/2020-27/11/2020

II. Administrative and scientific coordinators

Local coordinator
Name: Marcos
Firstname: Aboubakar
Gender: Male
Position : Professor
Institution: Institut de Mathématiques et de Sciences Physiques (IMSP)
Country: Benin
Email address:****@yahoo.fr

External coordinator
Name: Seara
Firstname: Tere
Gender: Female
Position: Professor
Institution: Universitat Politecnica de Catalunya
Country: Spain
Email address:****@upc.edu

III. Description of the project

Scientific content
The school aims to introduce graduate students and young researchers to the modern theory of
dynamical systems and its applications, with a focus on geometric, topological, and numerical

methods. The courses will be focused on some key topics in low and high dimensional dynamics, including Hamiltonian systems and Celestial Mechanics. They will also discuss statistical properties of dynamical systems, and their relation with the underlying geometric structures. The participants will be provided with an introduction to the basic material and with the necessary background, before proceeding with the more advanced topics. Geometric and topological methods in dynamics provide a global picture of the structures that organize the dynamics. They are also essential in designing and implementing rigorous numerical experiments. The courses will survey a range of applications including classical mechanics, space mission design, dynamical astronomy, material science and biology. Besides lectures, we are also planning sessions devoted to solving exercises and to computer experiments.

Host institution and local context in mathematics

The Institut de Mathématiques et de Sciences Physiques (IMSP) is a graduate school of mathematics and physics, located at Dangbo in Benin. The creation of IMSP goes back to 1988, and is a result of the initiative of the late Professor Abdus Salam, Nobel Prize winner and former Director of the International Centre for Theoretical Physics (ICTP), in Trieste (Italy), with the approval of the government of Benin. The fundamental goal of IMSP is to enhance training and research in Mathematics, Physics and Computer Sciences in Africa, mainly in the Sub-Saharan Africa. In this framework, IMSP since 1988, has selected and trained talented students for Master and PhD in PDEs, Control theory, Riemannian, pseudo-Riemannian and Symplectic Geometries, in Physics, in Mathematical Physics and in Computer Science, for 15 countries of Africa. More than 82 PhD have been delivered and all those alumni have obtained a position in their home institutions or in some foreign ones. Many seminars, schools and conferences have been organized by the institution. Many of those activities have been partially funded by the CIMPA, and recently from 7 to 19 July, 2014, IMSP hosted the CIMPA-UNESCO-BENIN research school on Algebraic Number theory and Applications.

The Institut de Mathématiques et de sciences Physiques has be recognized by the African Mathematics Union (UMA) as a scientific centre of excellence in 1994 and next, by the DAAD-Germany through his program: Scholarships for postgraduate Training and Research at African Re gional Networks of Centers of Excellence. In 2013, the Institute has been selected and funded as African Centre of Excellence (ACE) by the World bank in order to promote regional specialization in areas that address regional challenges of development and strengthen its capacity to deliver quality training and applied mathematics.

Prior work related to the project and motivation

A series of international activities on Dynamical Systems at IMSP has taken place at IMSP since September 2015 under the coordination of Prof. Stefano Luzzatto (ICTP, Trieste, Italy).

The objective has been to give a precise and formal mathematical foundation to the Theory of Dynamical Systems and Ergodic Theory, which constitute a really active and vibrant area of mathematical research and have strong interactions with many different areas of research from geometry to functional analysis, from mathematical physics to applications in natural sciences and modeling.

The first series of Dynamical Systems courses took place during the period from the 28th September to the 4th December 2015 and consisted of the following series of courses : « Introduction to Dynamical Systems » by Sergey Tikhomirov (Max Planck Institute for Mathematical Sciences, Leipzig, Germany), « Introduction to Dynamical Systems and Ergodic Theory » by Samuel Senti (Federal University of Rio de Janeiro, Brazil), « Introduction to Complex Dynamics » by Marcos Abate (University of Pisa, Italy) and « Introduction to Ergodic Theory » by Stefano Luzzatto (ICTP, Trieste, Italy).

The second series of Dynamical Systems courses started from the 30th of May 2016 and went on

until the 24th of June 2016, it involved mainly students from IMSP. It was about « Introduction to circle maps » with lectures given by the professors Marco Martens (Stony Brook, USA) and Liviana Palmisano (Polish Academy of Sciences, Poland). The next school of CIMPA in the same field will enable us to have more participants from our sub region.

Expected impact of the project

The expected impact is firstly to have IMSP students interested and involved in the Theory of Dynamical Systems and eventually choose a related research problem, and secondly to set up a Master/Doctoral Programme in the Field of Dynamical Systems. We also believe that this school will have a powerful impact on the development of mathematical sciences in the neighboring countries.

The course will train students in geometric, topological and numerical aspects of dynamical systems by means of, and in the context of applications. The proposed topics will integrate the knowledge from different areas of mathematics, and will provide a solid basis for scientific computing. The course will outline several new research directions and present the latest developments in the field. The range of applicability of the proposed topics, including particle physics, hydrodynamics, biological and medical sciences, space mission design, computer vision, will very likely motivate the participants to achieve a deeper knowledge and a solid understanding of the dynamical systems phenomena. The school participants will be engaged by the instructors through hands-on activities, practical sessions, problem solving, and research projects. They will be organized in study groups, and will be in charge of writing detailed notes. The lecturers in the program are planning to give a list of follow-up readings and some concrete problems that the students could try to do, and to organize a reading group which can meet using Skype or social media. The lecturers have also expressed their availability to further mentor and co-advice the participating students and fellows in their future academic endeavors.

Infrastructure

The school will take place on the campus of IMSP at Dangbo in the seminar room (20 to 25 participants), or in the auditorium for a more larger number of participants. Speakers will be accommodated in some hotel close to the campus and a shuttle between the hotel and the campus will be arranged free. CIMPA participants will share dormitories (double rooms) on the campus. Moreover, all lunches will be taken all together on the campus.

Expected participants

Among participants, we plan to have about about 30 students and researchers from Togo and 20 CIMPA participants from the subsaharian region. We may also have additional foreign participants not covered with CIMPA's support.

IV. Scientific committee

Member 1
Name: Seara
Firstname: Tere
Gender: Female
Institution: Universitat Politecnica de Catalunya
Country: Spain
Role: External coordinator and lecturer

Member 2
Name: De la Llave
Firstname: Rafael
Gender: Male
Institution: Georgia Institute of Technology
Country: USA
Role: Lecturer

Member 3
Name: Luzzatto
Firstname: Stefano
Gender: Male
Institution: ICTP
Country: Italy
Role: Lecturer and member of the math deparment of ICTP

Member 4
Name: Salvo
Firstname: Laura
Gender: Female
Institution: University Rome 3
Country: Italy
Role: Scientific coordinator of a network between Italy and West Africa

Member 5
Name: Gidea
Firstname: Marian
Gender: Male
Institution: Yeshiva University
Country: USA
Role: Lecturer

V. Organizing committee

Member 1
Name: Marcos
Firstname: Aboubakar
Gender: Male
Institution: IMSP
Country : Benin
Role: Logistics and local support of the school

Member 2
Name: Degla
Firstname: Guy
Gender: Male
Institution: IMSP
Country : Benin
Role: Logistics and local support of the school

Member 3
Name: Gangbo
Firstname: Wilfrid
Gender: Male
Institution: UCLA
Country : USA
Role: International liaison

Member 4
Name: Seara
Firstname: Tere
Gender: Female
Institution: Universitat Politecnica de Catalunya
Country: Spain
Role: Course curriculum and women mentoring

Member 5
Name: Salvo
Firstname: Laura
Gender: Female
Institution: University Rome 3
Country: Italy
Role: Fund raising from international institutions

VI. Scientific program

Introductory courses

Course 1
Title: Introduction to computation in dynamics
Duration: 5h
Lecturer's name: De la Llave
Lecturer's firstname: Rafael

Lecturer's gender: Male

Lecturer's institution: Georgia Institute of Technology

Lecturer's country : USA

Abstract of the course: The Poincare program for dynamical systems was based on finding invariant objects which acted as landmarks to understand the complicated behavior of systems. For mathematicians finding meant proving theorems of existence that also provided information about location and geometric properties. In modern days, this has also come to mean the use of computers. In this lectures, we plan to present some ideas on how to do computations that go hand in hand with the numerical understanding. Students will be encouraged to carry out some programming tasks in OCTAVE (a public domain computational package, with versions in almost all computer systems). There lectures will be followed by some practical tasks.

Course 2

Title: Computational Techniques, Tools and Implementations

Duration: 5h

Lecturer's name: Miranda

Lecturer's firstname: Josepha

Lecturer's gender: Female

Lecturer's institution: Universitat Politecnica de Catalunya

Lecturer's country: Spain

Abstract of the course: In this course we present some of the basic techniques that have been used to understand and take profit of the phase space of problems in the field of celestial mechanics and astronomy. Among other topics and extensions, the course includes computation of periodic orbits, and invariant tori using pure numerical or semianalytical approaches. The examples considered are chosen because of their practical interest in science and technology, but the scope of the tools presented are not limited to these particular fields.

Course 3

Title: A Combinatorial/Topological Theory of Nonlinear Dynamics

Duration: 5h

Lecturer's name: Mischaikow

Lecturer's firstname: Konstantin

Lecturer's gender: Male

Lecturer's institution: Rutgers University

Lecturer's country : USA

Abstract of the course: It is reasonably safe to assume that much of the work of this century in dynamics will involve the study of multiparameter multiscale systems, will be done using computational methods, and will be based on the analysis of large data sets. In the 70s C. Conley developed a purely topological framework for the study of dynamics. This course will describe this work but from an algorithmic perspective. In particular, it will be shown that the essential ideas of Conley theory can be recast in a combinatorial and algorithmic framework which leads to efficient novel computational methods for nonlinear systems. Furthermore, the use of algebraic topology and new computational topological tools allows one to draw mathematically rigorous conclusions even in the context of finite resolution (measurements) in both phase space and parameter space.

Course 1

Title: Invariant manifolds, splitting of separatrices and exponentially small phenomena

Duration: 5h

Lecturer's name: Seara

Lecturer's firstname: Tere

Lecturer's gender: Female

Lecturer's institution: Universitat Politecnica de Catalunya

Lecturer's country: Spain

Abstract of the course: In this course we will talk about invariant manifolds in dynamical systems. We will focus on hyperbolic invariant objects and their stable and unstable manifolds. We will explore the important phenomenum of splitting of separatrices, a problem that Poincare called the "fundamental problem of dynamics". We will present the classical Melnikov method to measure the distance between invariant manifolds in a perturbative setting. Finally we will explain some exponentially small phenomena.

Course 2

Title: Topological Methods for Detection of Instability and Chaos

Duration: 5h

Lecturer's name: Gidea

Lecturer's firstname: Marian

Lecturer's gender: Male

Lecturer's institution: Yeshiva University

Lecturer's country: USA

Abstract of the course: We will introduce some fixed point theory based methods to detect instability and chaotic behavior in dynamical systems. These methods can be used in both analytical arguments and rigorous numerical experiments. We will discuss some applications to Aubry-Mather theory, Hamiltonian instability, and celestial mechanics.

Course 3

Title: Introduction to Differentiable Ergodic Theory

Duration: 5h

Lecturer's name: Luzzatto

Lecturer's firstname: Stefano

Lecturer's gender: Male

Lecturer's institution: ICTP

Lecturer's country: Italy

Abstract of the course: I will give a brief but precise introduction to the the basic notions of Ergodic Theory, including definitions and examples of invariant and ergodic measures and their main properties. I will prove ergodicity of Lebesgue measure for one-dimensional full branch piecewise linear maps and, depending on the time available, discuss the generalizations to piecewise C^2 maps.

Training sessions

All the afternoons will be devoted to problem solving sessions and computer labs.

VII. Tentative schedule

Day 1 (Monday)
Morning: Courses (3h)
Afternoon: Problem solving sessions (3h)

Day 2 (Tuesday)
Morning: Courses (3h)
Afternoon: Computer labs (3h)

Day 3 (Wednesday)
Morning: Courses (3h)
Afternoon: free

Day 4 (Thursday)
Morning: Courses (3h)
Afternoon: Problem solving sessions (3h)

Day 5 (Friday)
Morning: Courses (3h)
Afternoon: Computer labs (3h)

Day 6 (Saturday)
Morning: free
Afternoon: free

Day 7 (Sunday)
Morning: excursion
Afternoon: excursion

Day 8 (Monday)
Morning: Courses (3h)
Afternoon: Problem solving sessions (3h)

Day 9 (Tuesday)
Morning: Courses (3h)
Afternoon: Computer labs (3h)

Day 10 (Wednesday)
Morning: Courses (3h)
Afternoon: free

Day 11 (Thursday)
Morning: Courses (3h)
Afternoon: Problem solving sessions (3h)

Day 12 (Friday)
Morning: Courses (3h)
Afternoon: Discussion with participants + Closing ceremony

VIII. Provisional budget (expenses)

Transportation costs: 20,000€
- Airfare of speakers: 7,000€
- Transportation of 20 CIMPA students: 8,000€
- Transportation of other foreign students and researchers :for all non-local students: 5,000€

Lodging: 3,600€
- Accomodation for speakers (hotel): 3,500€

Meals: 8,400€
- Midday meals for for all participants (60): 6,000€
- Dinners for CIMPA participants (20): 2,400€

Logistics: 1000€
- Opening ceremony, posters, tags, secretary

Others: 1000€
- Excursion

Total expenses : 34,000€

IX. Anticipated funding (resources)

Financial support requested from CIMPA: 12,000€

Local funding: 13,000€
- IMSP

International funding:
- AUF: 1000€
- IMU: 1000€
- All speakers will cover their travel expenses with their own grants (7,000€)

Total resources: 34,000€

X. Support and involvement of local institutions

The Université of Lomé provide accomodation for all CIMPA participants and some other participants. Moreover, the université provides a shuttle for the speakers. The funding given by IMSP comes from the support provided by the Word Bank.

XI. Additional remarks and comments