COMPLEX ANALYSIS

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1. How to apply?

The videos and notes of the courses will be available to everyone. However in order to be able to have interactions with each student as in a real class, we will also set up a chat platform and organize video sessions. We therefore want to limit the number of participants to 50 young participants from developing countries. This is why you need to apply if you want to enjoy the full experience.

Please note that online teaching and learning require a great commitment: do not apply if you are not ready and available to devote time to study and participate actively to the classes.

If needed, selected candidates can benefit from CIMPA support to acquire a good connection or buy small equipment to participate to the courses in good conditions.

2. How will the course take place?

Starting March 15, we will release a series of videos which will contain the material of the courses in addition to notes. You will have time to study these videos and courses till April. In order to assist you during that time, a chat platform will be set up so that you can ask your questions to the lecturer and also work in groups. At the same time, exercises will be suggested so that you can learn by doing (best way!) and video tutorial sessions will be organized so that you can present your solutions in turn.

3. Description of the courses

In this Master-level course I propose to introduce some topics in Complex Analysis. The course will be more profitable for studends having already studied some topics in elementary complex analysis, such as Holomorphic functions, Cachy theory and residue theory. Nevertheless, in the first 5-6 lectures I will introduce these basic topics, mostly without proofs, in order thar a wider range of potential students could follow the course. These preliminaries are contained in the first four chapters of [2], for instance, but these chapters include more material that what I need. Particularly, I would recommend to have a look to Sections 2.1 (Integration on Paths), 2.2 (Power Series), 2.4 (Further Applications), Chapter 3 (The general Cauchy Theorem), 4.1 (Singularities) and 4.2 (Residue Theory).

The course will consist in 25-30 hours. We will cover, tentatively, the following topics:

1. Basics of Complex Analysis (5 hours).
   - 1.1. Holomorphic functions.
   - 1.2. Integration along paths.
   - 1.3. Cauchy Theorem in a Disk. Analytic Functions. Applications.
   - 1.4. Logarithm, argument, index.
   - 1.5. Notice of the general Cauchy Theorem.
2. Residues and applications (3 hours).
   - 2.1. Singularities. Residues.

2.2. Open mapping Theorem. Rouché Theorem. Argument Principle. Hurwith Theorem.
3. Conformal mappings (8 hours).
   3.1. Stereographic projection.
   3.2. Automorphisms of $\bar{\mathbb{C}}$. Möbius transformations.
   3.3. Automorphisms of $\mathbb{C}$, D. Schwarz Lemma.
   3.4. Families of analytic Functions. Montel Theorem.
   3.5. Riemann Mapping Theorem.
4. Infinite Products (4 hours).
   4.1. Infinite Products of complex numbers.
   4.2. Infinite products of analytic functions.
   4.3. Weierstrass Products. Factorization of Analytic Functions.
5. Gamma and Zeta Functions (4 hours).
   5.1. Gamma and Zeta Functions. Definitions and Properties.
   5.2. The Prime Number Theorem.
6. Elliptic Functions (4 hours).
   6.1. Periodic meromorphic functions.
   6.2. Weierstrass elliptic function.
   6.3. Connections with the theory of elliptic curves and Riemann surfaces (notice).

References

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