Course title:

Convex analysis in geodesic spaces

Course summary:

In this course, we will go through the development of a young area of convex analysis in geodesic spaces, especially the CAT(k) spaces. This space can be thought of as the nondifferentiable counterpart of Riemannian manifolds. Riemannian manifolds of certain curvatures, metric trees, cubical complexes, and the space of phylogenetics are typical examples of a CAT(k) space. The theory flourished with an attempt to generalize fixed point theory for nonexpansive maps outside of a linear space, followed by the study of the proximal algorithm of (geodesic) convex functions that enriches the study of abstract gradient flows. Further development of convex subdifferentials and monotone vector fields are then addressed, and a few interesting applications will be presented.

References:

1. Bacák, Miroslav. *Convex analysis and optimization in Hadamard spaces*. Vol. 22. Walter de Gruyter GmbH & Co KG, 2014.
2. Ambrosio, Luigi, Nicola Gigli, and Giuseppe Savaré. *Gradient flows: in metric spaces and in the space of probability measures*. Springer Science & Business Media, 2008.
3. Gigli, Nicola, and Francesco Nobili. "A differential perspective on Gradient Flows on ${\sf CAT}(\kappa) $-spaces and applications." *arXiv preprint arXiv:2012.12952* (2020).
4. Chaipunya, Parin, Fumiaki Kohsaka, and Poom Kumam. "Monotone vector fields and generation of nonexpansive semigroups in complete CAT (0) spaces." *Numerical Functional Analysis and Optimization* (2021): 1-30.