CIMPA Research in Pairs 2025 (Report)

I. Information

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Name:	FOTUE TABUE Alexandre
Home Institution:	University of Bertoua (Cameroon)
Total duration of stay:	From 1st May to 10 July 2025 (70 days)
Host colleague:	Edgar Martinez-Moro, University of Valladolid (Spain)

II. Activity Report

From early May to 10 July 2025, I conducted a productive 70-day research visit at the Institute of Mathematics of the University of Valladolid (IMUVa), hosted by Prof. Edgar Martinez-Moro. My primary research focus was on additive complementary pairs (ACPs) of cyclic codes over finite commutative rings. Collaborating closely with Prof. Martinez-Moro, we investigated the structural properties of these codes, which are crucial building blocks for constructing quantum error-correcting codes. Our collaborative research successfully established the freeness

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of such ACPs under specific conditions, formulated a characterization theorem, and constructed an explicit example over a finite chain ring. These results were compiled into the preprint *Additive Complementary Pairs of Additive Cyclic Codes over Finite Chain Rings*, submitted to *Finite Fields and Their Applications* and available on arXiv: `https://arxiv.org/abs/2506.10381`. Subsequently, we completed and submitted another article, entitled *Duality on Group Algebras over Finite Chain Rings: Applications to Additive Group Codes*, to the *Transactions of the American Mathematical Society* and available on arXiv: `https://arxiv.org/abs/2508.07461`. This latter work investigates additive group codes derived from group algebras over finite chain rings. In its first part, we analyze the underlying ring extensions and establish several module isomorphisms that yield a decomposition of group algebras, thus offering a structural characterization of the associated additive group codes. In the second part, we construct a symmetric, nondegenerate traceEuclidean inner product on the group algebra and examine

additive complementary pairs in this context. For two-sided ACPs, we prove that the orthogonal complement of one code with respect to this duality coincides precisely with the image of the other code under a specific involutive anti-automorphism of the group algebra. This result reveals a profound connection between ACP theory, orthogonal direct-sum decompositions in module theory, representation theory, and the structural properties of group algebras over finite chain rings.
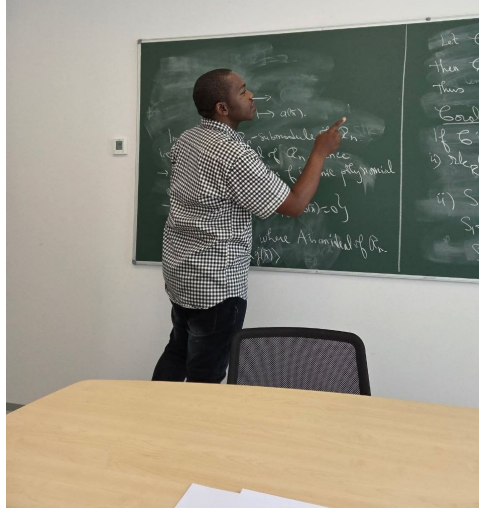
Research discussion with Prof. Martinez-Moro

A key component of the visit included a trip to the Autonomous University of Barcelona from June 1st to 6th to record my lecture "*Finite Rings and their Applications in Coding Theory*," further disseminating our field's foundations.

Future developments:

- Possibility of establishing joint supervision of master's and doctoral students between the two institutions;

- Prepare a CIMPA School 2028;

- Extend current results to quantum error-correcting codes.

III. Feedback

The programme was exceptionally organized. Suggestions for improvement:

- During the stay, encourage and support participation in conferences and seminars held in the host country and neighboring regions.;

- Consider extended stays (90 days) for complex projects.

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